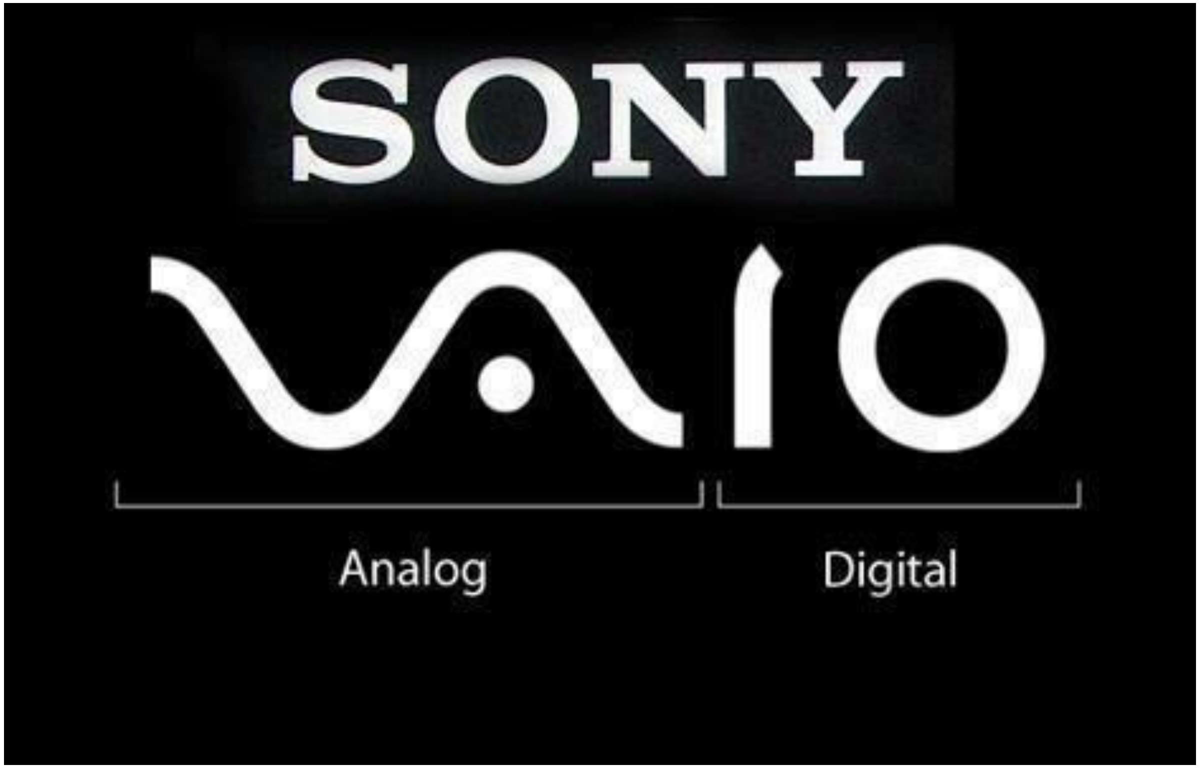
Sony VAIO Logo



The Sony VAIO logo illustrates the integration of analog and digital technology. The VA letters form an analog wave and the IO part represents a binary one and zero.

Principles of Communications
ECS 332

Dr. Prapun Suksompong
prapun@siit.tu.ac.th

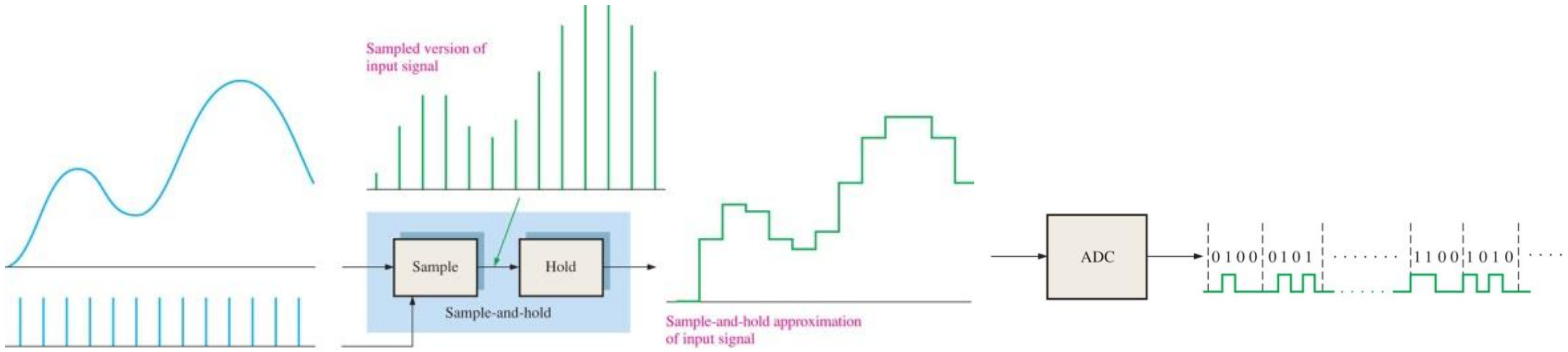
8. Sampling and Alising

Office Hours:
BKD 3601-7

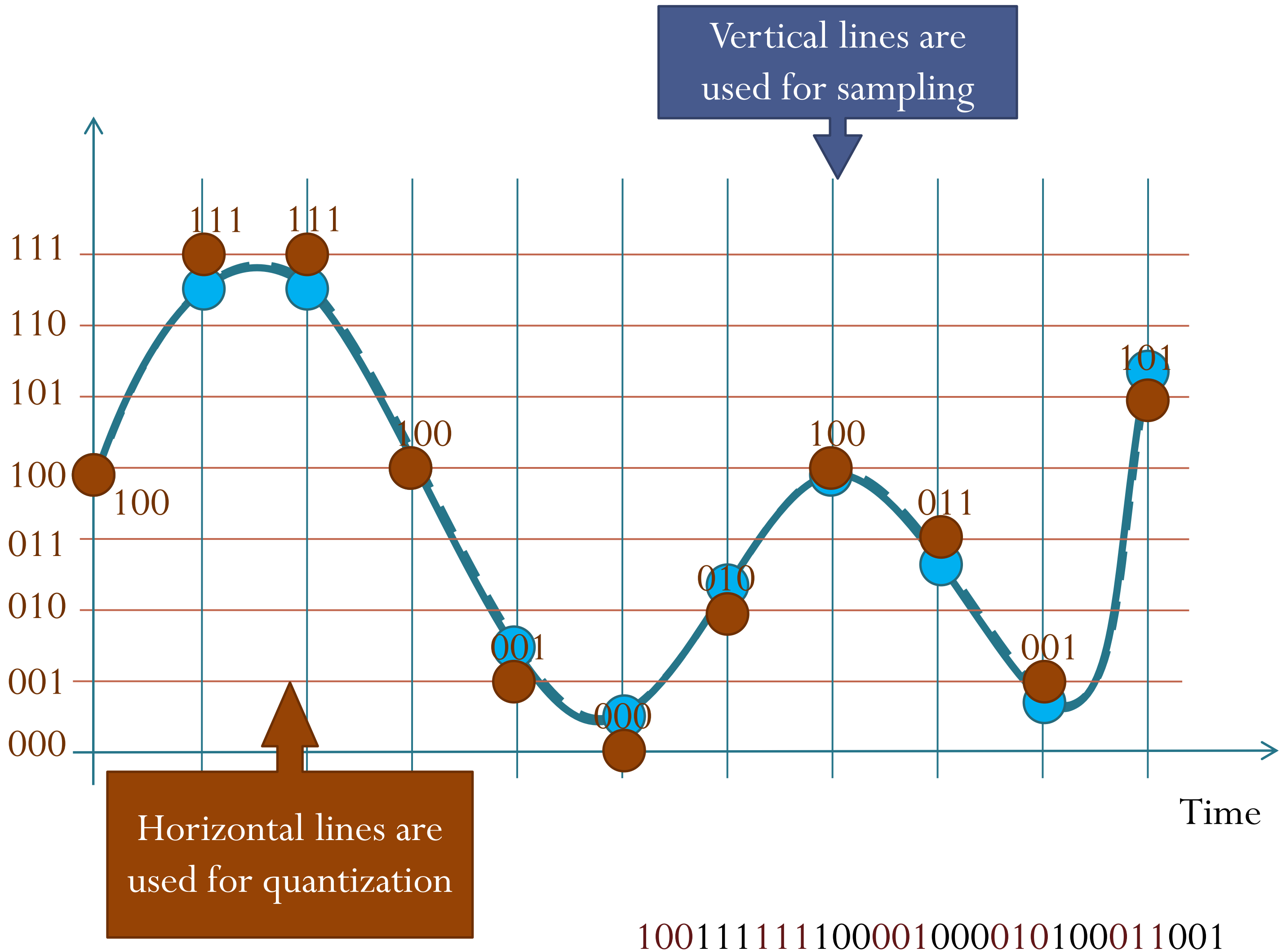
Monday	14:40-16:00
Friday	14:00-16:00

Converting Analog Signals to Digital

- The real world is analog!
- Interfacing between analog and digital is important.
- Digitization
  1. **Sampling (and hold)**: Discretize the time
     - Get sampled values of the analog signal.
  2. **Quantization**: Discretize quantity values
     - Convert each sampled value to a binary code.



Digitization (analog to digital)



Sampling = loss of information?

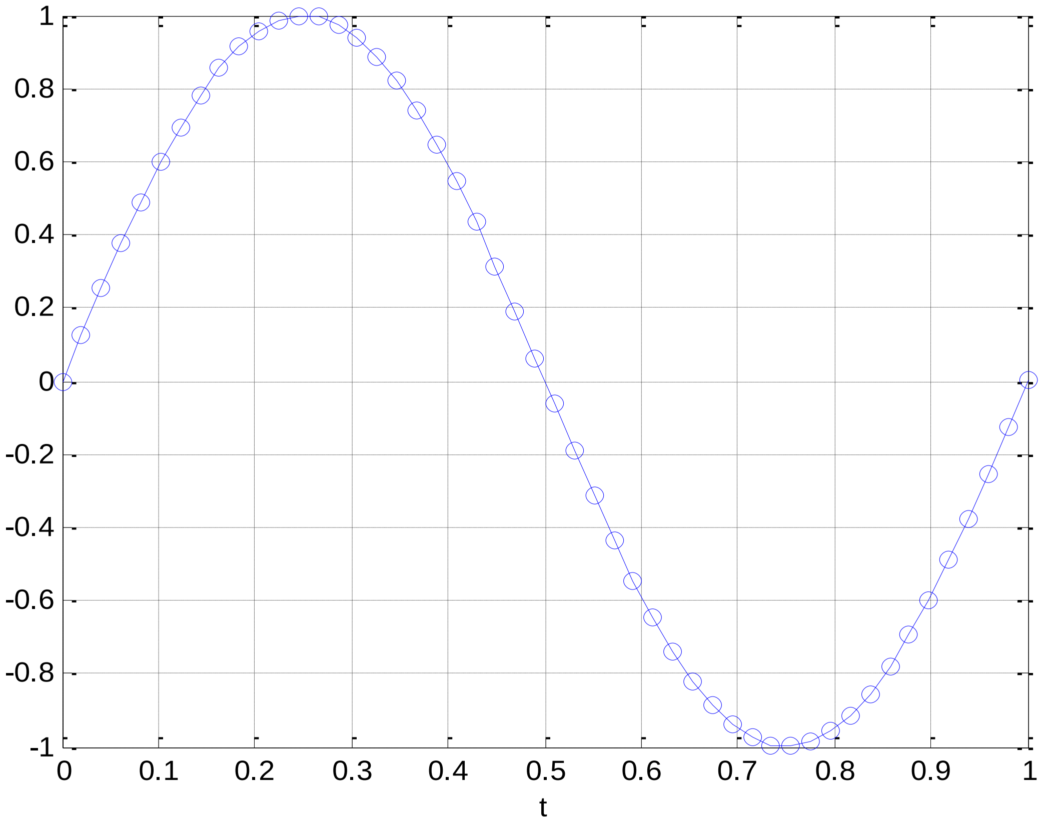
- At first glance, digitization of a continuous signal (audio, image) appears to be an enormous loss of information, because a continuous function is reduced to a function on a grid of points.
- Therefore the crucial question arises as to which criterion we can use to ensure that the sampled points are a valid representation of the continuous signal, i.e., there is no loss of information.

Sampling

- **Sampling** is the process of taking a (sufficient) number of discrete values of points on a waveform that will define the shape of wave form.
- Suppose that we sample a signal at a uniform rate, once every T_s seconds.
  - We refer to T_s as the **sampling period**, and to its reciprocal $f_s = 1/T_s$ as the **sampling rate**.
- The more samples you take, the more accurately you can define a waveform.
  - *Caution*: If the sampling rate is too low, your may experience distortion (**aliasing**).

Example: sin(100πt) (1/4)

This is the plot of sin(100πt). What's wrong with it?



[AliasingSin_2.m]

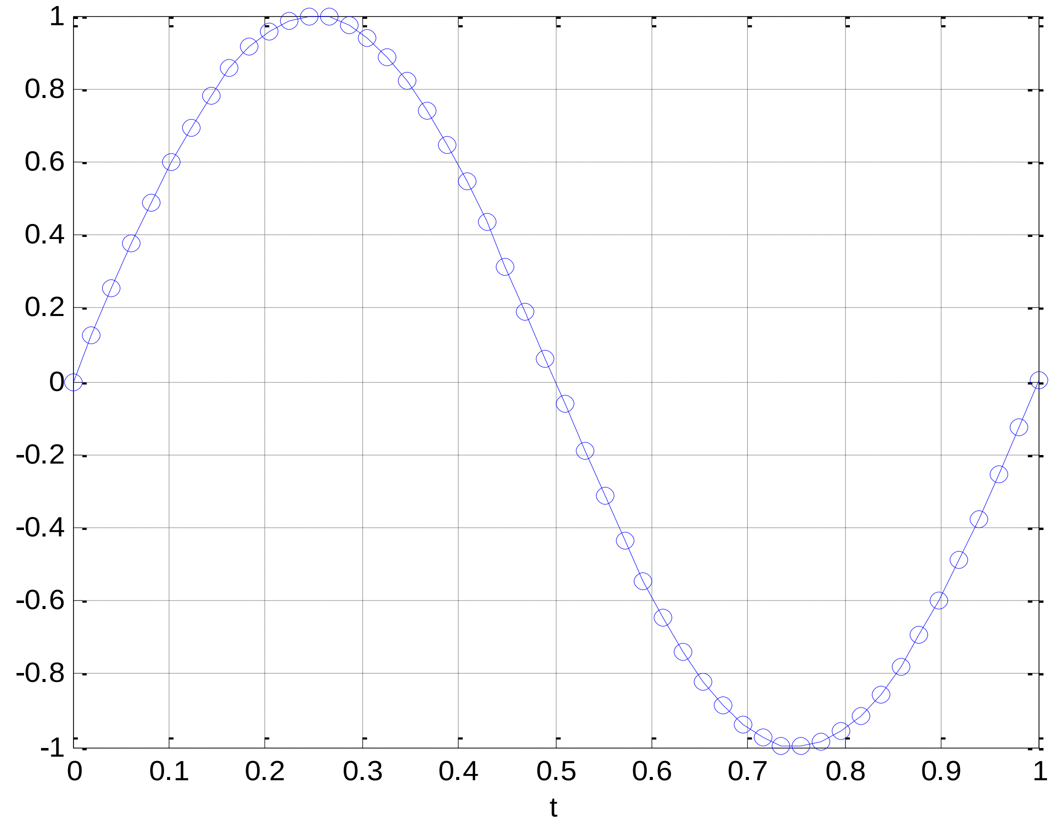
Example: sin(100πt) (2/4)

Signal of the form $\sin(2\pi f_0 t)$ have frequency $f = f_0$ Hz.

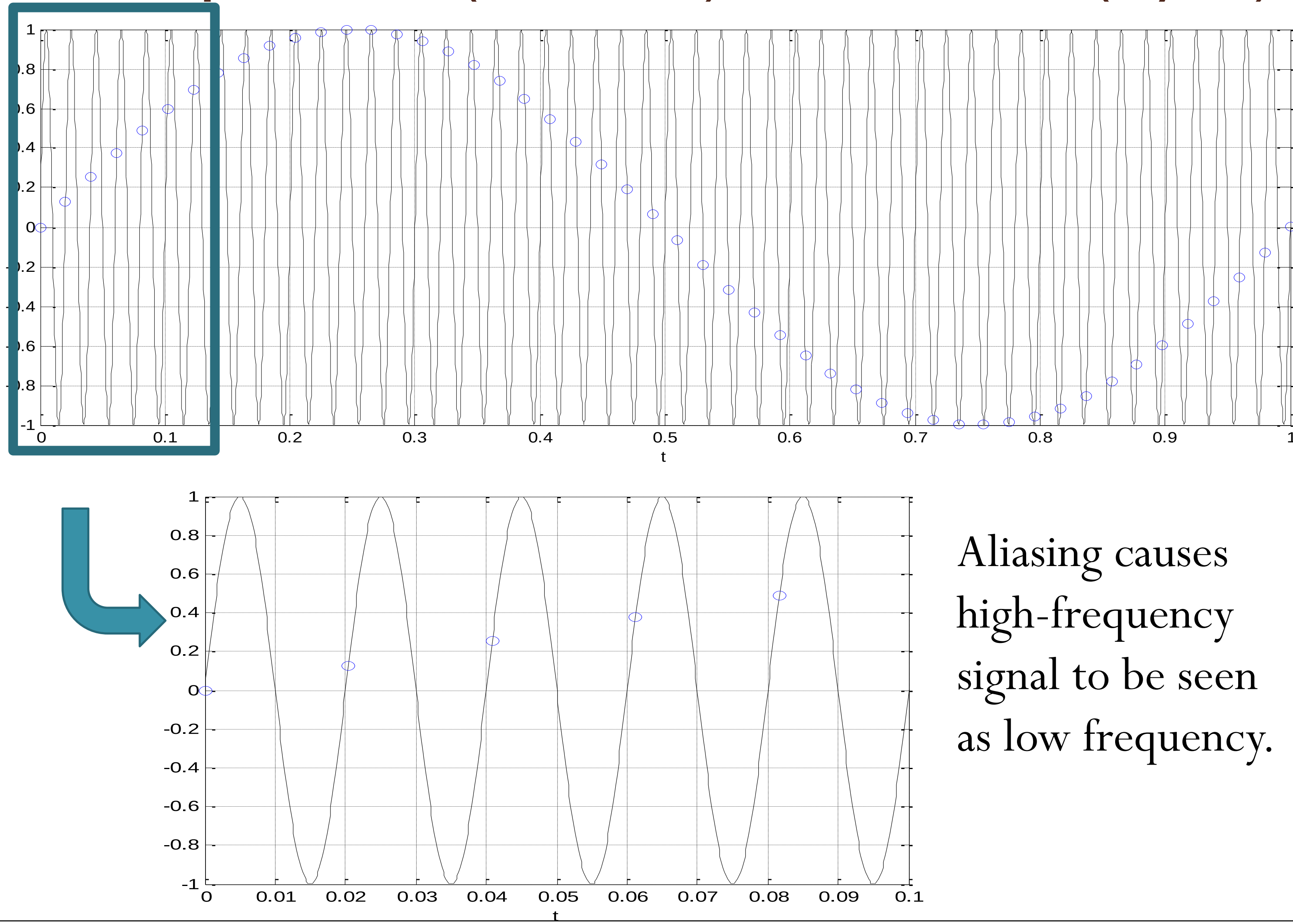
So, the frequency of $\sin(100\pi t)$ is 50 Hz.

From time 0 to 1, it should have completed 50 cycles. However, our plot has only one cycle.

It looks more like the plot of $\sin(2\pi t)$



Example: sin(100πt) (3/4)



Aliasing causes high-frequency signal to be seen as low frequency.

Sampling Theorem

- In order to (correctly and completely) represent an analog signal, **the sampling frequency, f_s, must be at least twice the highest frequency component of the analog signal**.
- Given a sampling frequency, f_s, the **Nyquist frequency** is defined as $f_s/2$.
- Given that highest (positive-)frequency component $f_{\max}$ of the analog signal, the **Nyquist sampling rate** is $2f_{\max}$ And the **Nyquist sampling interval** is $1/(2f_{\max})$

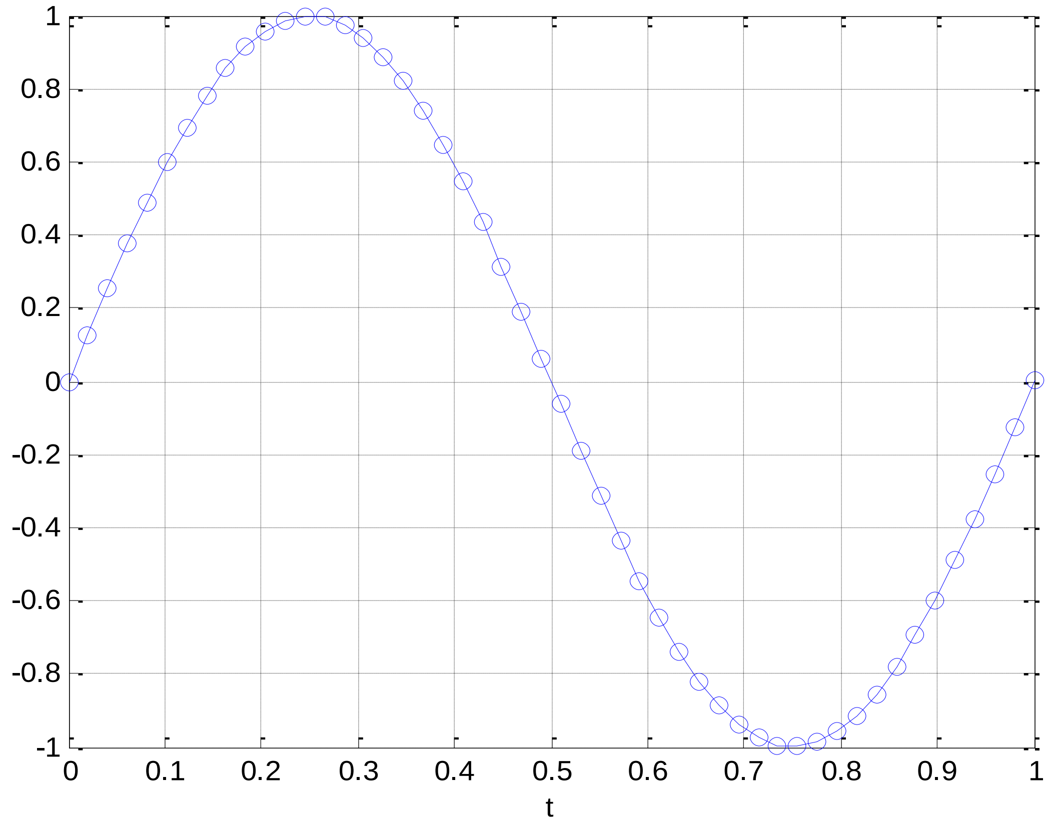
Example: sin(100πt) (4/4)

Signal of the form $\sin(2\pi f_0 t)$ have frequency $f = f_0$ Hz.

So, the frequency of $\sin(100\pi t)$ is 50 Hz.

We need to sample at least 100 times per time unit.

Here, the number of sample per time unit is 49, which is too small to avoid aliasing.



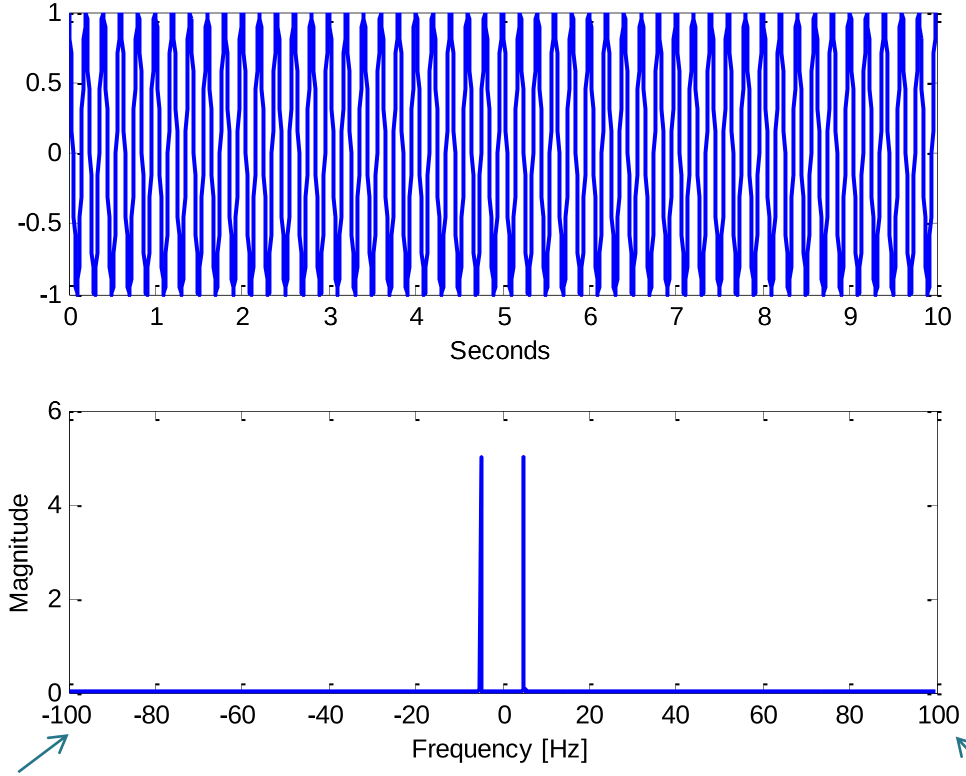
Ex: Aliasing

- If you've ever watched a film and seen the wheel of a rolling wagon appear to be going backwards, you've witnessed ***aliasing***.

plotspec.m

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(5)t)$

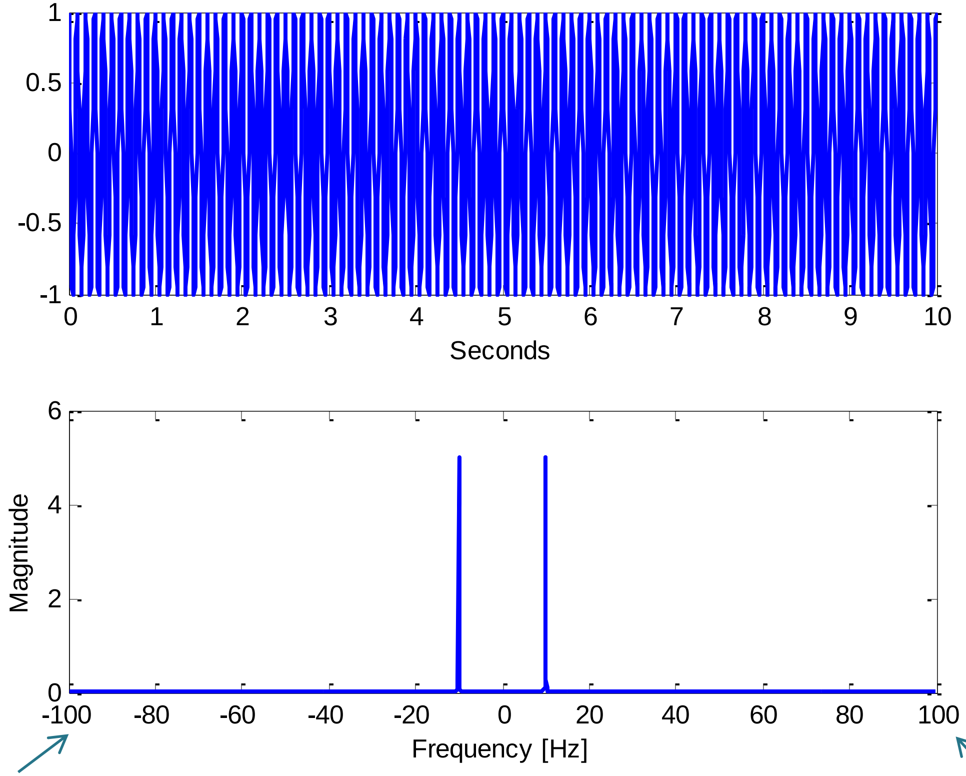


$-\frac{f_s}{2}$ $\frac{f_s}{2}$

plotspec.m

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(10)t)$



$-\frac{f_s}{2}$

$\frac{f_s}{2}$

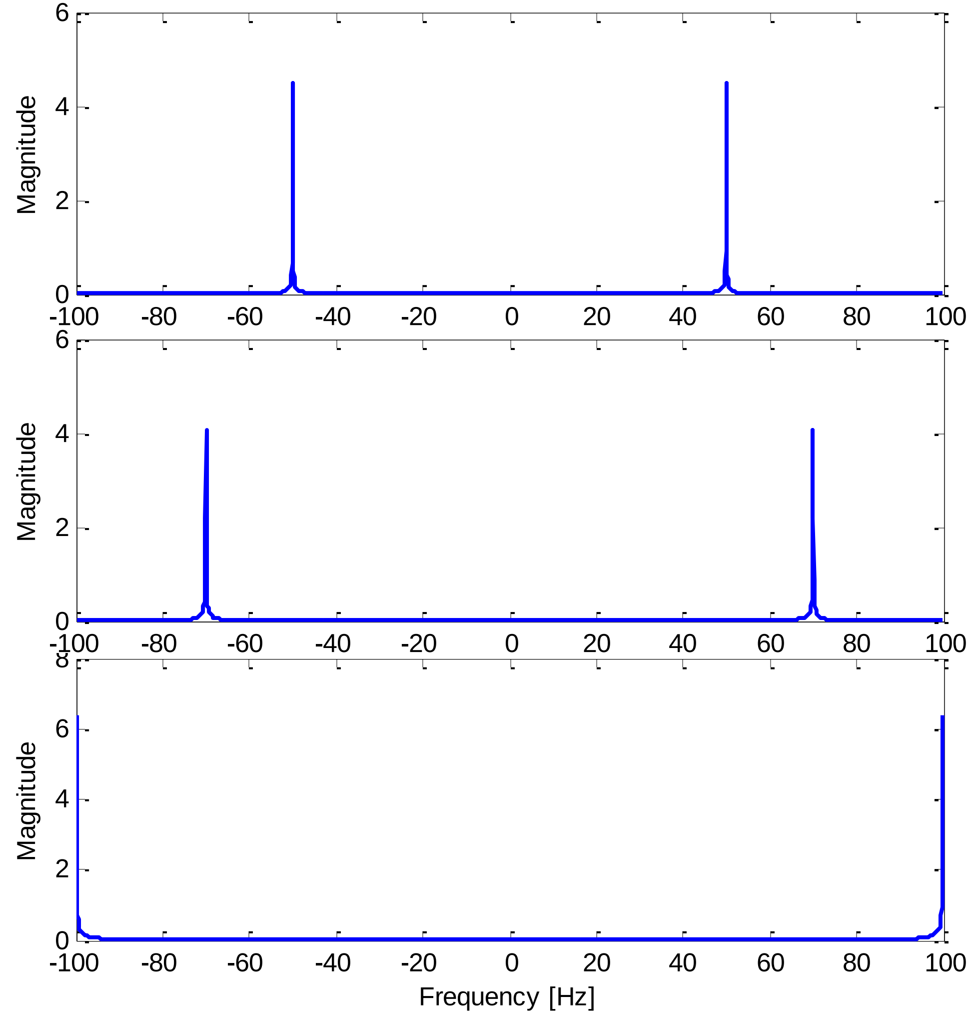
plotspec.m

- f_s: Sampling frequency = 200 samples/sec

$\cos(2\pi(50)t)$

$\cos(2\pi(70)t)$

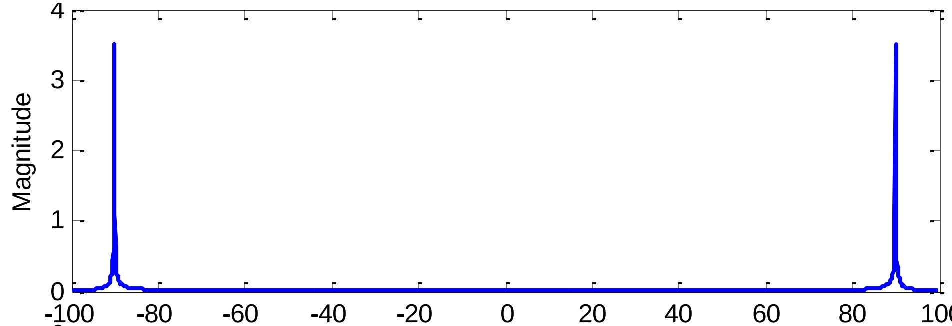
$\cos(2\pi(100)t)$



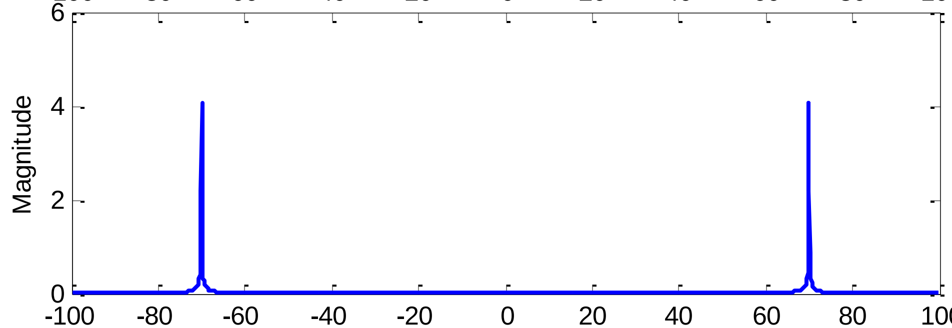
plotspec.m

- f_s: Sampling frequency = 200 samples/sec

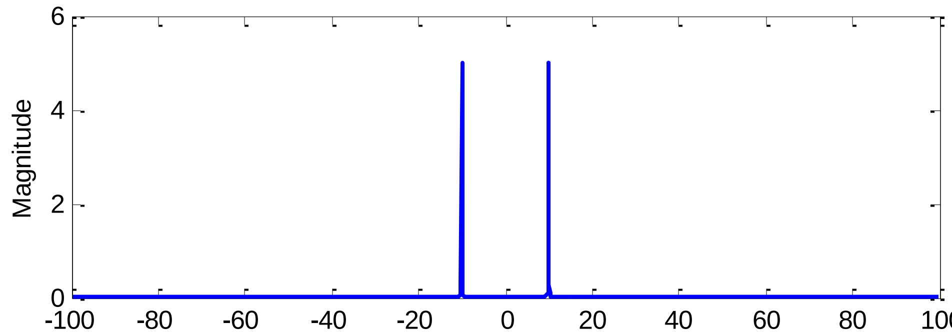
$\cos(2\pi(110)t)$



$\cos(2\pi(130)t)$



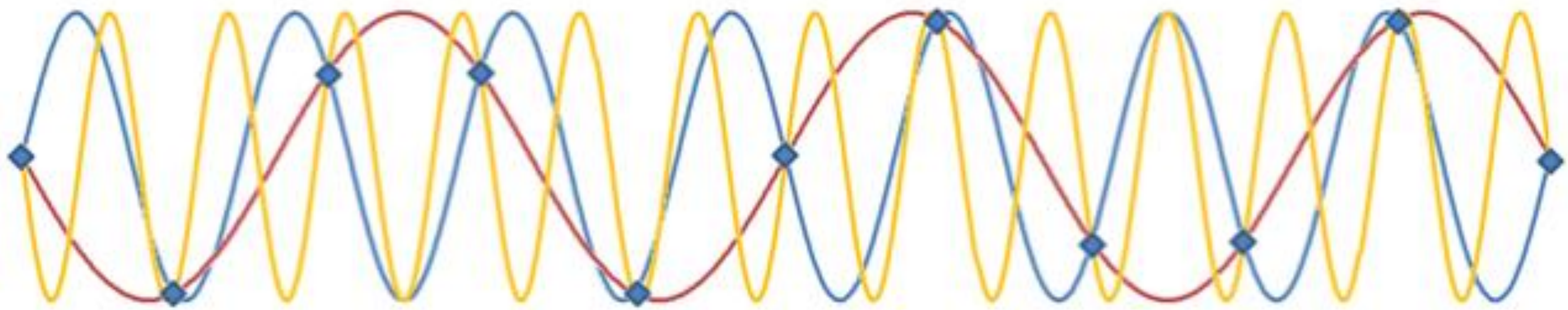
$\cos(2\pi(190)t)$



Pac Man

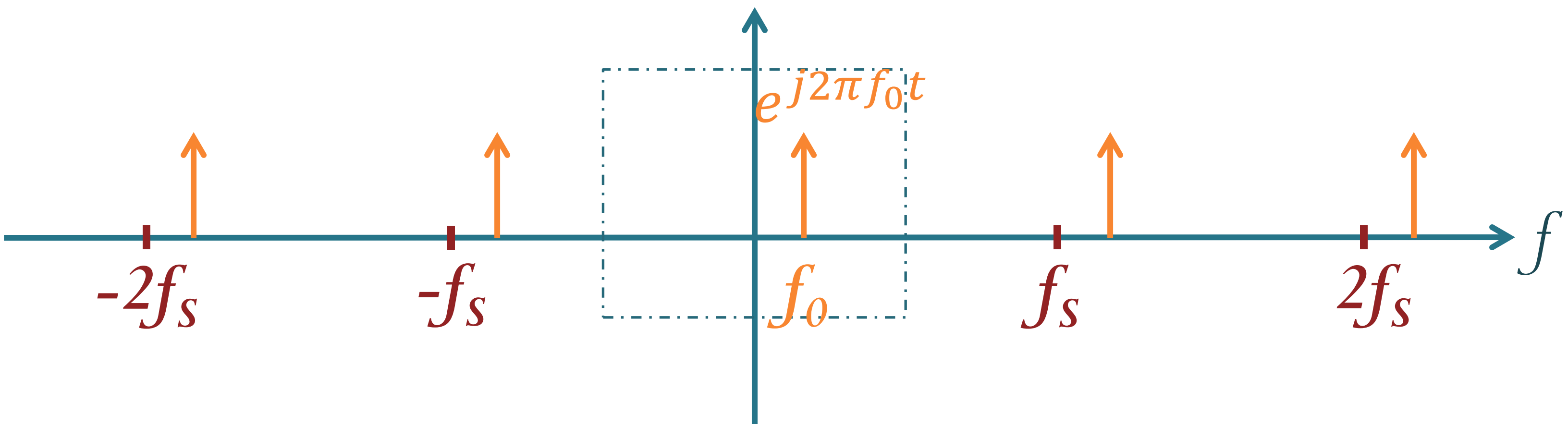
Another Example

- When sampled at 10 Samples per sec, there is no way to tell the difference between 3Hz, 7Hz, or the 13Hz waves below.

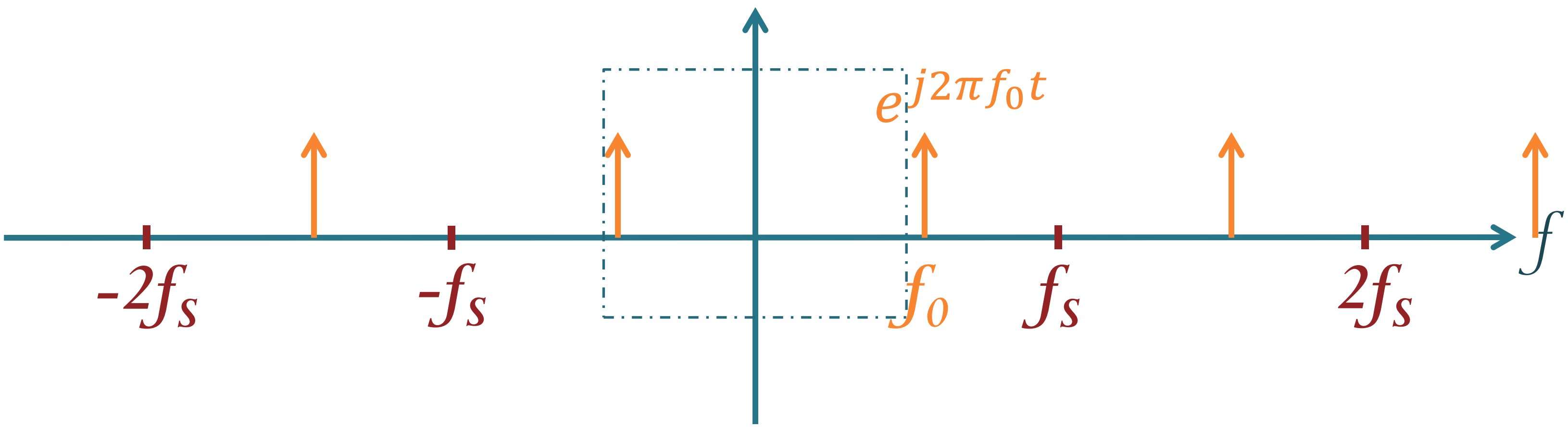


Aliasing: One complex exponential

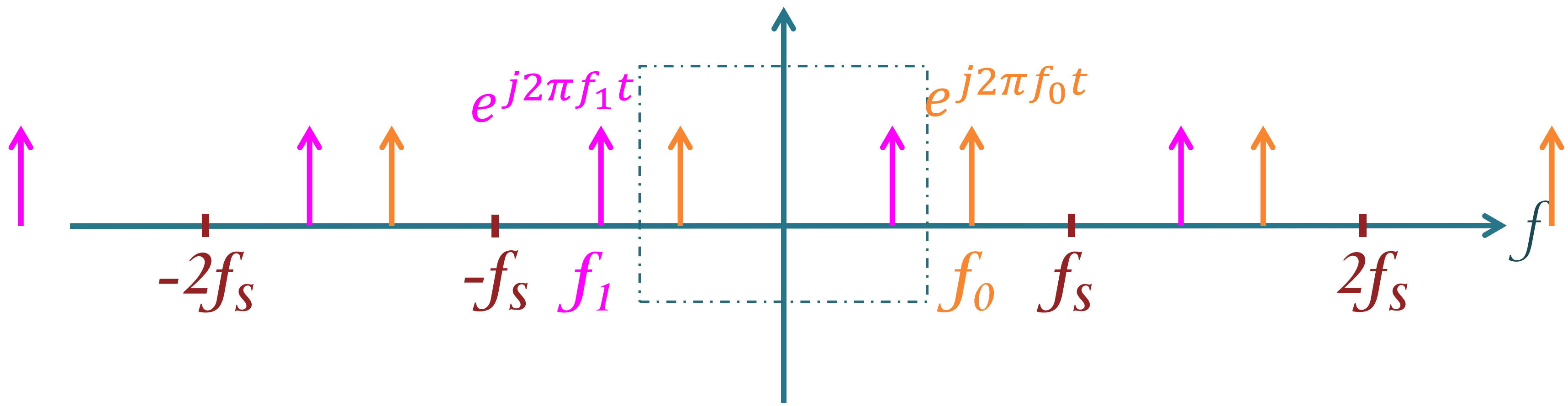
Let's increase f_0



Aliasing: One complex exponential



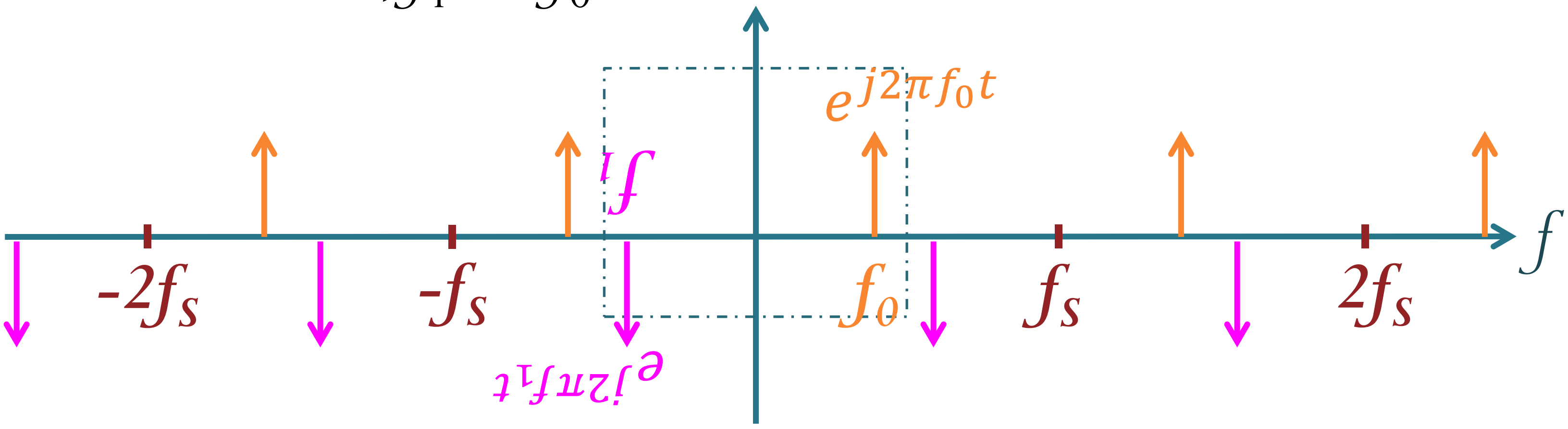
Aliasing: Two complex exponentials



Aliasing: Two complex exponentials

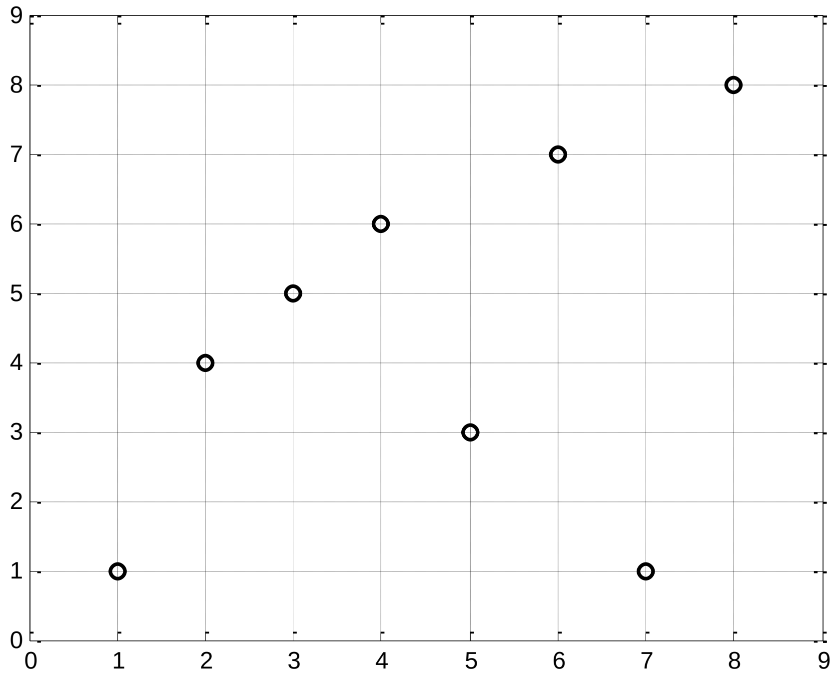
Let's change f_0 and f_1.

For sine wave, $f_1 = -f_0$.

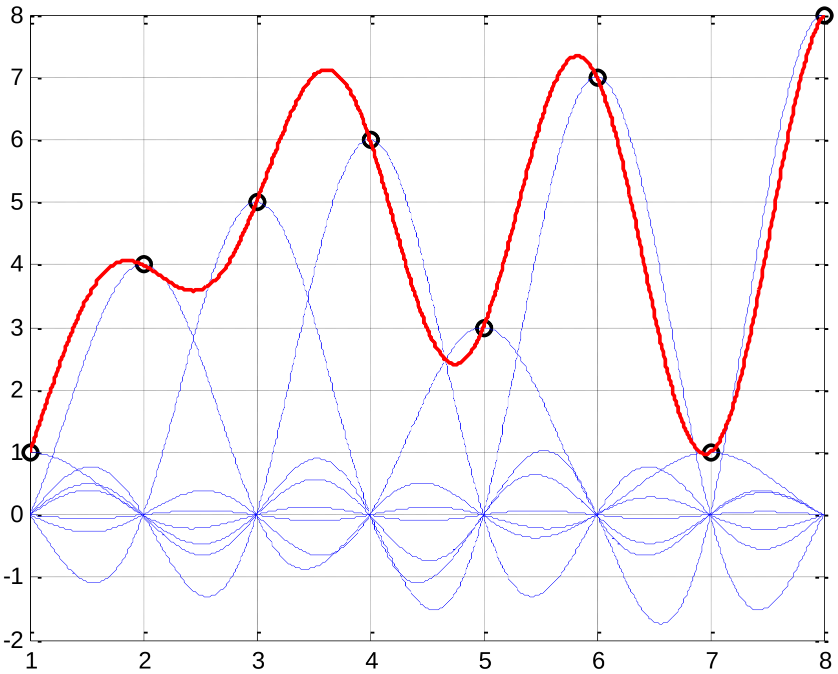


Reconstruction

m[k]



m(t)



Practical Reconstruction